

## Comparison of spectral decomposition methods

**John P. Castagna, University of Houston, and Shengjie Sun, Fusion Geophysical discuss a number of different methods for spectral decomposition before suggesting some improvements possible with their own variation of ‘matching pursuit’ decomposition.**

In seismic exploration, spectral decomposition refers to any method that produces a continuous time-frequency analysis of a seismic trace. Thus a frequency spectrum is output for each time sample of the seismic trace. Spectral decomposition has been used for a variety of applications including layer thickness determination (Partyka et al, 1999), stratigraphic visualization (Marfurt and Kirlin, 2001), and direct hydrocarbon detection (Castagna et al., 2003; Sinha et al., 2005).

Spectral decomposition is a non-unique process, thus a single seismic trace can produce various time-frequency analyses. There are a variety of spectral decomposition methods. These include the DFT (discrete Fourier Transform), MEM (maximum entropy method), CWT (continuous wavelet transform), and MPD (matching pursuit decomposition). None of these methods are, strictly speaking, ‘right’ or ‘wrong’. Each method has its own advantages and disadvantages, and different applications require different methods. The DFT and MEM involve explicit use of windows, and the nature of the windowing has a profound effect on the temporal and spectral resolution of the output. In general, the DFT is preferred for evaluating the spectral characteristics of long windows containing many reflection events, with the spectra generally dominated by the spacing between events. The MEM is often difficult to parameterize and may produce unstable results.

The CWT is equivalent to temporal narrow-band filtering of the seismic trace and has an advantage over the DFT for broad-band signals in that the window implicit in the wavelet dictionary is frequency dependent. The CWT has a great disadvantage, however, in that the wavelets utilized must be orthogonal. The commonly used Morlet wavelet, for example, has poor vertical resolution due to multiple side lobes. Furthermore, for typical seismic signals, the implicit frequency dependent windowing of the CWT is not particularly important, and experience has shown that a DFT with a Gaussian window of appropriate length produces almost the same result as a CWT with a Morlet wavelet. MPD (Mallat and Zhang, 1993) is a more computationally intensive process than the others, but, as will be shown in this paper, it has superior temporal and spectral resolution if a compact mother wavelet is utilized.

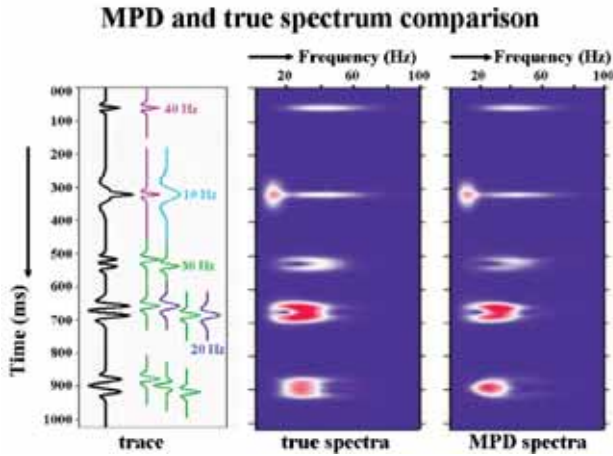
Matching pursuit decomposition involves cross-correlation of a wavelet dictionary against the seismic trace. The projection of the best correlating wavelet on the seismic trace is then subtracted from that trace. The wavelet dictionary is then cross-correlated against the residual, and again the best correlating wavelet projection is subtracted. The process is

repeated iteratively until the energy left in the residual falls below some acceptable threshold. As long as the wavelet dictionary meets simple admissibility conditions, the process will converge. Most importantly, the wavelets need not be orthogonal. The output of the process is a list of wavelets with their respective arrival times and amplitudes for each seismic trace. The inverse transform is accomplished simply by summing the wavelet list and the residual, thus reconstructing the original trace. The wavelet list is readily converted to a time-frequency analysis by superposition of the wavelet frequency spectra. Simple matching pursuit has difficulty in properly determining the precise arrival time of interfering wavelets – usually it will slightly misplace the wavelets which will also result in a slightly incorrect wavelet center frequency. Also, it can be seen that the process is path dependent: a slight change in the seismic trace may result in an entirely different order of subtraction. Thus, it may result in lateral instability of the non-unique time-frequency analyses. Cross-correlation of the wavelet dictionary against the seismic trace is essentially a continuous wavelet transform, so it can be seen that the method involves iteratively performing hundreds, if not thousands, of wavelet transforms for each seismic trace.

In this paper, we utilize a variation of matching pursuit called exponential pursuit decomposition (EPD). The method treats complex interference patterns as containing ‘gravity wells’ at the correct wavelet locations, and the selected wavelet location is iteratively attracted to the correct location. The profound advantage of EPD over other methods is that there is no windowing, and corresponding spectral smearing. The spectra for reflections from isolated interfaces that can be resolved by the method are the same as the seismic wavelet producing those reflections. The method can thus be used with confidence for direct hydrocarbon indication and stratigraphic visualization for thin beds.

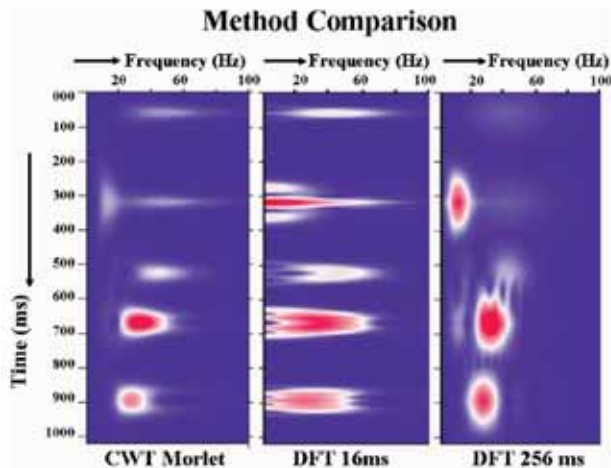
The classical Heisenberg Uncertainty Principle states that the product of temporal and frequency resolution is constant. One must normally pay the price of decreasing resolution in one domain, to increase resolution in the other. In EPD, there is no windowing and it is the bandwidth of the digital seismic data that limits resolution, not the windowing process. Thus, the Heisenberg Uncertainty Principle does not come into play. As a result, EPD provides better temporal AND spectral resolution than the other methods. In comparing spectral decomposition methods, it is important to keep in mind what the goal of the analysis is.

There is no such thing as a ‘right’ or ‘wrong’ spectral decomposition. The real question is whether the method being



**Figure 1** Synthetic trace obtained by adding wavelets of with different center frequencies, true time-frequency amplitude analysis, and MPD time-frequency amplitude spectra. The superposed wavelets are placed according to their arrival time and center frequency is indicated by the wavelet color (light blue = 10 Hz, dark blue = 20 Hz, green = 30 Hz, lavender = 40 Hz). The spectra color bar is from blue (zero amplitude) to red (strongest amplitude).

applied is ‘useful’ or ‘not useful’ for the specific application. If the objective is determining bed thickness for resolvable reflections, lower vertical resolution methods that do not resolve the individual reflections are needed. Thus, for thick bed thickness determination, the DFT will do the job, while EPD spectra will not be affected by bed thickness for resolved tops and bases. If one is interested in the spectral characteristics of individual reflectors (to measure attenuation for example) the reflector of interest must be isolated by the decomposition method. We will show that EPD is more likely to resolve individual reflectors than other decomposition methods. On the other



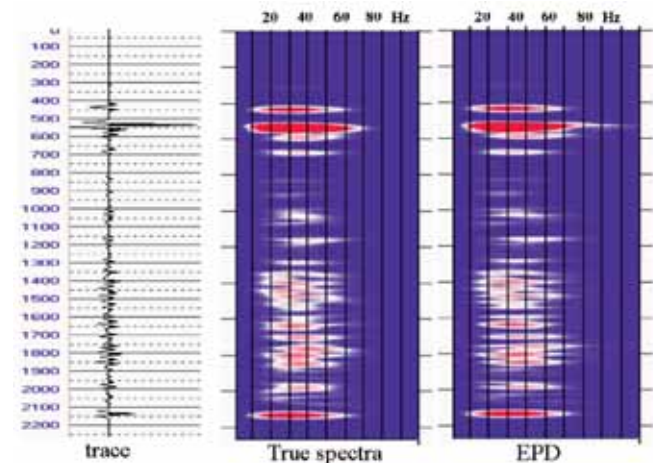
**Figure 2** Time-frequency analysis for seismic trace shown in Figure 1 including continuous wavelet transform (CWT) with Morlet wavelet, and Discrete Fourier Transform with 16 msec and 256 msec boxcar windows.

hand, one may be interested in making inferences about stratigraphic patterns for interfaces whose spacings are well below the conventional tuning thickness, and which produce a composite reflection. In this case, the EPD spectra can be used for sub-tuning stratigraphic visualization, while DFT spectra may be dominated by interferences with other discrete reflections within the analysis window.

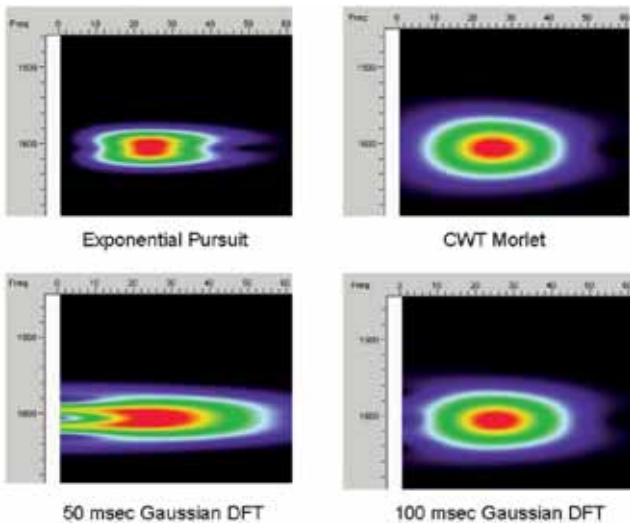
### Synthetic examples

The only circumstance where the ‘true’ time-frequency decomposition is known, is for the case of synthetic data. Recalling the Fourier Superposition Principle, if we know the arrival times and amplitudes of wavelets summed to make a synthetic seismic trace, and we know the spectra of those wavelets, we can determine the frequency spectrum as a function of time by simply summing time shifted and amplitude weighted wavelet spectra. In the case of a synthetic seismogram, we know the seismic wavelet spectrum, and the arrival times and amplitude weighting are defined by the reflection coefficients. It is instructive to compare spectral decomposition methods to the true time-frequency analysis for synthetic traces of various kinds.

In Figure 1, an unusual seismic trace is constructed for tutorial purposes. The trace consists of a superposition of wavelets with differing centre frequencies. The first seismic event (near 50 msec) is an isolated wavelet with a centre frequency of 40 Hz. Notice that the duration of its true spectrum is no greater than the duration of the wavelet in time. The second seismic event (near 300 msec) consists of a 40 Hz wavelet and a 10 Hz wavelet arriving at the same time. The true spectrum is bimodal, with peaks at 10 Hz and 40 Hz. The remaining seismic events occurring at later times are composite reflections for two or three closely spaced reflector arrival times. The true spectra are relatively smooth as the spectral notches of the interfering reflections are outside the bandwidth of the data. The temporal maxima of the spectra correspond to the arrival times of the interfering wavelets. The MPD time-frequency analysis is very similar to the true time-frequency analysis. Notice that for



**Figure 3** Comparison of true spectra and exponential pursuit decomposition (EPD) for a realistic synthetic seismic trace.

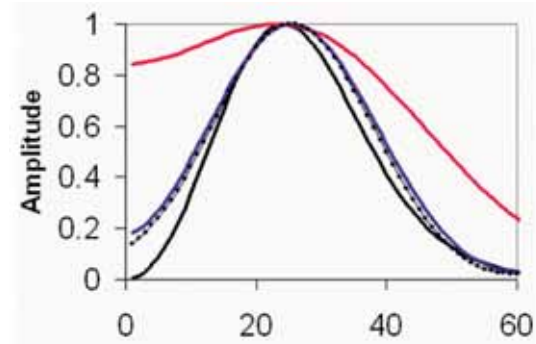


**Figure 4** Time-frequency analyses for a layer with equal and opposite reflection coefficients and a time thickness of 20 milliseconds. The CWT is performed with a Morlet wavelet dictionary, and the DFT is performed with Gaussian windows of 50 and 100 milliseconds.

isolated events, the time duration of the MPD spectra are the same as the true spectra. Note also that the bi-modal spectrum for the event near 300 msec is well resolved by MPD, and that the time duration of the spectral peaks persist in time for the duration of the wavelets.

Although the MPD time-frequency analysis is similar to the true time-frequency analysis for interfering events, there are noticeable differences. In particular, symmetrical events on the trace may be asymmetrical as a function of time for the MPD time-frequency analysis. This has to do with the path-dependence of the method. EPD minimizes the path-dependence and returns virtually the true time-frequency analysis for this simple example (not shown here as it is indistinguishable from the true spectrum).

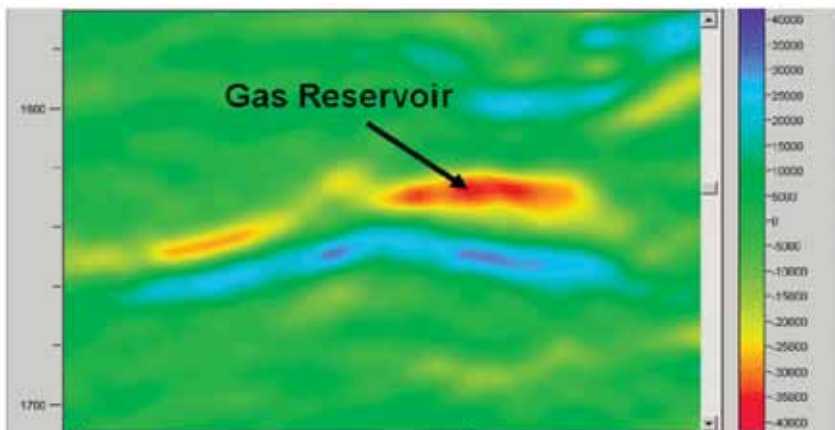
Figure 2 shows the inadequacy of the CWT and DFT methods. The CWT performed with a Morlet wavelet dictionary has limited temporal resolution and cannot separate



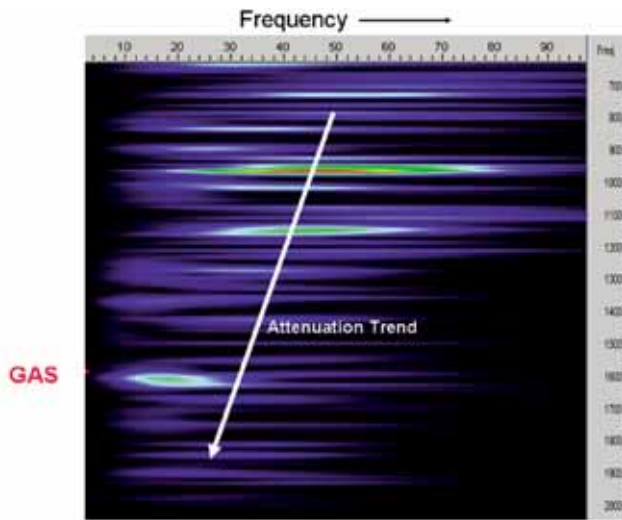
**Figure 5** Line spectra at a time of 1596 milliseconds for the single layer spectral decompositions shown in Figure 4. Curves shown include DFT with a 50 millisecond Gaussian window (red), DFT with a 100 millisecond Gaussian window (blue), CWT with a Morlet wavelet dictionary (dotted), and exponential pursuit decomposition (black solid).

the closely spaced events as well as MPD or EPD. A short window is required for the DFT to equal the resolution of MPD. Notably, the DFT with a 16 millisecond boxcar window widens the frequency spectra (a consequence of the Heisenberg Uncertainty Principle) thereby introducing energy at zero frequency that was not present in the original trace and producing false events associated with event side lobes. A 256 millisecond boxcar window smears the events out over time for the length of the window and introduces notches into the time-frequency spectra associated with time-spacing between events. Clearly, for this case, MPD and EPD are more useful for capturing individual seismic event characteristics that may be of interest for quantitative seismic analysis.

Application of EPD to a realistic synthetic seismogram (Figure 3) shows a remarkable capturing of the significant spectral characteristics of the true spectrum. In our experience no other technique can achieve this combination of temporal and frequency resolution. For this reason, we refer to spectral decomposition performed using MPD or EPD methods as instantaneous spectral analysis; meaning that the spectra given in the time-frequency analysis are indicative of the local spectral characteristics of the data without undesirable window-



**Figure 6** Stacked seismic section showing a bright spot caused by a low impedance Gulf of Mexico gas reservoir exhibiting a trough/peak response. The vertical axis is time in milliseconds.



**Figure 7** Time-frequency analysis using exponential pursuit for a seismic trace containing a gas reservoir. The white line illustrates the progress reduction of the peak frequency due to attenuation. The gas reservoir has anomalously low peak frequency.

ing-related effects such as spectral widening and notching.

A detailed look at the response of various spectral decomposition methods for closely spaced reflectors is shown in Figures 4 and 5 which are derived from a conventional synthetic seismogram calculated for a single layer using a 25 Hz Ricker wavelet. In Figure 4, it can readily be seen that EPD has the best combination of temporal and spectral resolution. It is interesting to note that the CWT with a Morlet wavelet dictionary is very similar to the DFT with a 100 millisecond Gaussian window. The differences in frequency resolution are

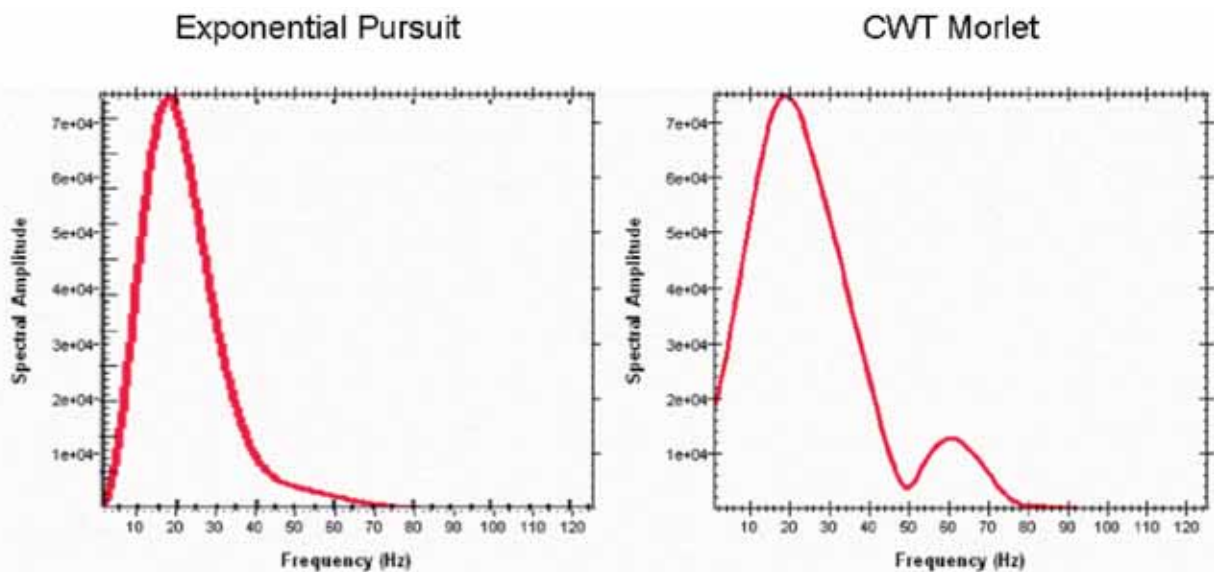
best seen on a graph of line spectra at a single time (Figure 5). The DFT and CWT methods widen the estimated spectrum while EPD yields the true wavelet spectrum.

### Real data example of exponential pursuit decomposition

Figure 6 shows a classical bright spot gas reservoir from the Gulf of Mexico shelf. Exponential pursuit decomposition produces a time frequency gather that shows the progressive drop in peak frequency with record time caused by attenuation, and also shows the abnormally low frequency of the reservoir reflection (Figure 7). Only EPD resolves the top and base of this reservoir. As seen in Figure 8, the CWT spectrum clearly shows a notch at 50 Hz, which corresponds to a layer thickness of about 20 milliseconds. The EPD line spectrum shown in Figure 8 does not exhibit this notch effect. For a variety of reasons, gas reservoirs are commonly lower frequency than the corresponding downdip brine filled rock. When this occurs, the ratio of the normalized downdip spectrum divided by the normalized gas reservoir spectrum may be diagnostic. As shown in Figure 9, the EPD spectral ratio shows a steady increase with increasing frequency, indicating a lack of high frequencies for the gas reservoir. On the other hand, the layer thickness notch at 50 Hz makes this ratio misleading when the CWT is used for spectral decomposition.

### Conclusions

Matching and exponential pursuit decomposition (MPD and EPD) do not involve windowing of the seismic data and thus have the best combination of temporal and spectral resolution as compared to the discrete fourier transform (DFT) and the continuous wavelet transform (CWT). EPD is laterally more stable than MPD and more accurately locates wavelets in time when wavelets interfere. Synthetic seismograms reveal the



**Figure 8** Comparison of spectra at a time corresponding to the top of the gas reservoir shown in Figure 6. The notch at 50 Hz for the CWT spectrum is a consequence of interference with the base reflector about 20 milliseconds below the top.

remarkable ability of EPD to separate interfering wavelets. A real data example for a Gulf of Mexico bright spot shows that the abnormally low frequency content of the reservoir is better revealed by EPD (as compared to CWT) because it is

less affected by interference between top and base reflectors. We conclude that EPD is the most useful method for quantitative analysis of the spectral characteristics of individual reflections.

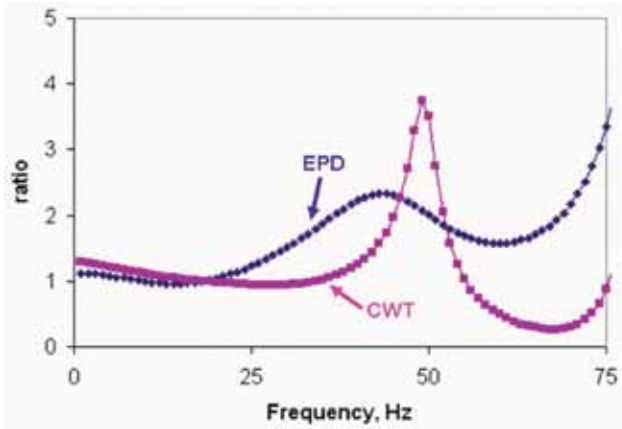


Figure 9 Ratio of down-dip spectra and gas reservoir spectra using exponential pursuit decomposition (EPD) and continuous wavelet transform with a Morlet wavelet (CWT).

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