Inverse spectral decomposition

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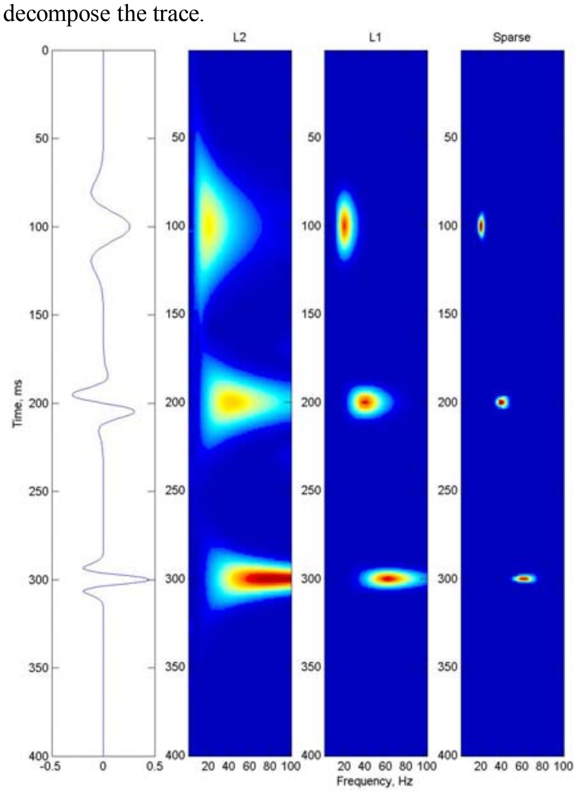
Summary

This paper introduces a method which spectrally decomposes a seismic trace by solving an inverse problem. In our technique, the reverse wavelet transform with a library of complex wavelets serves as a forward operator. The inversion reconstructs the wavelet coefficients that represent the seismic trace and satisfy an additional constraint. The constraint is needed as the inverse problem is non-unique. We show synthetic and real examples with three different types of constraints: 1) minimum L2 norm, 2) minimum L1 norm, and 3) sparse spike, or minimum support constraint. The sparse-spike constraint has the best temporal and frequency resolution. While the inverse approach to spectral decomposition is slow compared to other techniques, it produces solutions with better time and frequency resolution than popular existing methods.

Introduction

Spectral decomposition is a transformation that characterizes spatiotemporal variability in seismic data. The spectral decomposition attributes effectively differentiate both lateral and vertical lithologic and pore-fluid changes. Spectral decomposition is particularly successful in delineating stratigraphic traps and identifying subtle frequency variations caused by hydrocarbons. Thus, spectral decomposition research has gained considerable momentum in recent years. A number of techniques have been studied, such as the Continuous Wavelet Transform and Matching Pursuit Decomposition (Chakraborty and Okaya 1995, Castagna et al, 2003), and the Discrete Fourier Transform (Marfurt and Kirlin, 2001). The multitude of existing methods signifies the non-unique nature of spatiotemporal transformation. Hence, the search for the most convenient and most seismically revealing transformation actively continues.

This paper introduces a new transformation where we achieve spectral decomposition by solving an inverse problem. Namely, we minimize the objective functional which is a weighted sum of a misfit and a stabilizer. In this process, the reverse wavelet transform with a library of complex wavelets serves as a forward problem. The inversion reconstructs the wavelet coefficients that 1) represent the seismic trace, and 2) satisfy the constraint. The second criterion is needed since the transformation is non-unique. The space of coefficients spans time-frequency domain, thus it has two dimensions Nf and Nt, where Nf is number of frequencies and Nt is number of samples in the trace. The seismic trace has only one dimension (Nt). So, the inverse problem is grossly underdetermined, and hence non-unique. That is, there is more than one way to decompose the trace.



The figure above illustrates this point. The left panel shows a synthetic trace with three wavelets, with frequencies of 20, 40 and 60 Hz and phases of 0, 90 and 0 degrees respectively. The next panel shows inverse decomposition with minimum L2 norm constraint, the second from the right panel shows the decomposition with minimum L1 norm constraint, and the right panel shows the sparse decomposition. These plots display only the amplitude, the phase is not shown, though it is readily calculated. All three decompositions represent the trace, in a sense that composing the wavelets with the coefficients from any of three distributions matches the trace. Mathematically, there is no way to say that one solution is better than the other. The only way to differentiate is to apply these transformations to many practical cases and then subjectively judge which one is more useful for interpretation purposes.

Arguably, the minimum L2 norm solution looks similar to results of other known methods (such as CWT) that produce smooth distributions. The sparse spike solution has superior resolution in time and frequency. The minimum L1 norm solution is less resolved but could be more robust in practice than the sparse solution. All three solutions produce phase, as an additional useful attribute.

Theory

Our theory follows the conventional inverse problem logic. We denote the trace as *d* (the data), the reverse wavelet transform as *F* (the forward modeling operator) and the wavelet coefficients as *m* (the model). Then, the mathematical statement of the problem is the minimization of the Tikhonov parametric functional:

$$\|real(Fm)-d\|^2+\alpha S(m)=\min$$

The first term in this equation is the misfit, which is responsible for matching the decomposition with the data. The second term is the constraint, which shapes the resulting distribution of coefficients. Factor α is called the regularization parameter.

We use a library of complex wavelets (translated to all times of the trace) to compose operator *F*. Hence, the solution *m* is a complex quantity which has both amplitude and phase. The choice of the wavelet library (and hence the choice of forward operator *F*) is a critical factor in determining the utility of the spectral decomposition. The question of choice of the constraint *S(m)* may be even more important due to the tendency of the constraint to influence the solution even more than *F* does, especially for underdetermined problems. So, we focus on *S(m)*.

There are a multitude of choices for *S*. Along with the traditional minimum L2 norm constraint

$$S_{L2}(m)=\sum_{i=1}^{N_f N_t} m_i^2 ,$$

that produces smooth and poorly resolved solutions. We find minimum L1 norm and sparse spike constraints to be particularly interesting. The minimum L1 norm constraint is given by the following formula:

$$S_{L1}(m)=\sum_{i=1}^{N_f N_t}|m_i|.$$

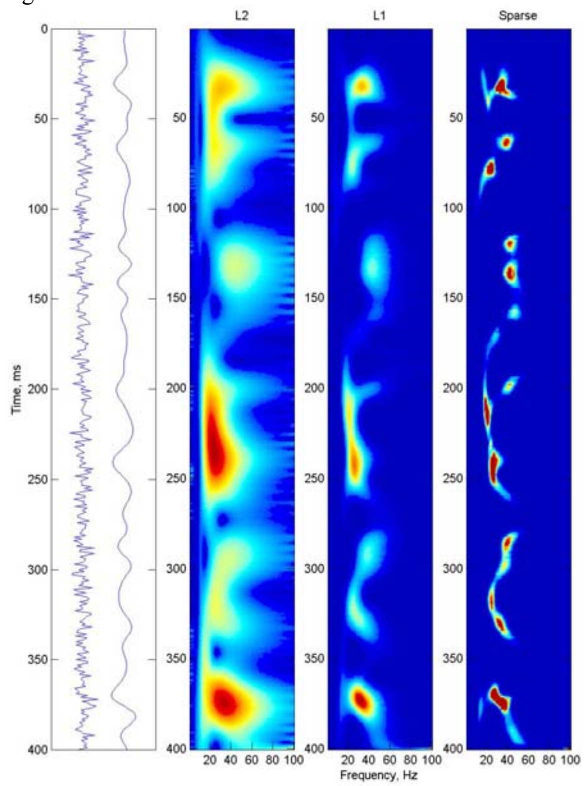
And the sparse spike constraint is given as:

$$S_{L0}(m)=\sum_{i=1}^{N_t N_f}\frac{|m_i|^2}{|m_i|^2+\beta^2}.$$

where $\beta=10^{-8}\max(m)$ is a small number, related to machine precision. Note that the sparse spike constraint has a minimum where the distribution of *m* has the smallest number of non-zeros, i.e. sparse distribution. It is similar to the minimum support constraint used in (Portniaguine and Zhdanov, 1998, 2002). Except, here we use amplitudes of complex values rather than the scalars. Since the numerical minimization technique for these types of functionals (re-weighted conjugate gradient relaxation) is also described in (Portniaguine and Zhdanov, 1998, 2002), we do not discuss it here. We only note that the re-weighted relaxation is relatively time consuming, which makes inverse decomposition methods slow. However, there is hope of significantly speeding it up by incorporating compression into the algorithm (Portniaguine and Zhdanov, 2002).

Realistic trace example

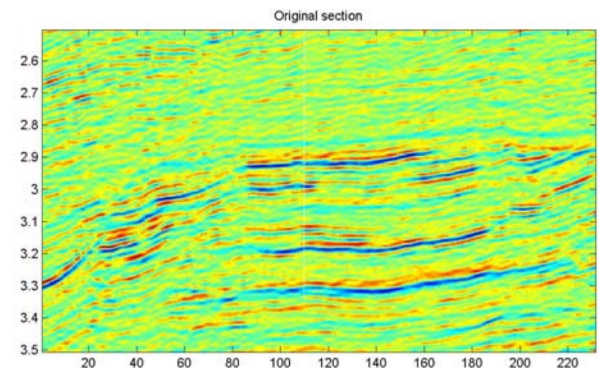
We present two examples of the technique. One is the decomposition of the realistic seismic trace, shown in the Figure to the bottom.

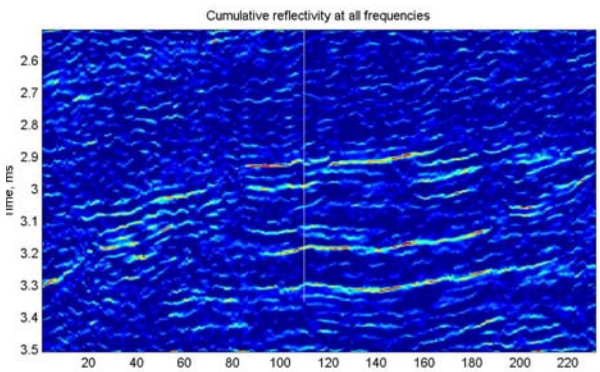


The leftmost panel shows the simulated reflectivity (Gaussian noise) and the synthetic trace, produced by convolving 30 Hz Ricker wavelet with the reflectivity. The next three panels show minimum L2 norm, minimum L1 norm and sparse decompositions, respectively. We can see that spiky decomposition has much higher frequency and time resolution. The decomposition result exhibits oscillations of the base wavelet frequency around 30 Hz, depending on the local reflectivity spectrum. While advantages of one technique over the other are subjective, they clearly charactarize different levels of details in the seismic data.

Real data example

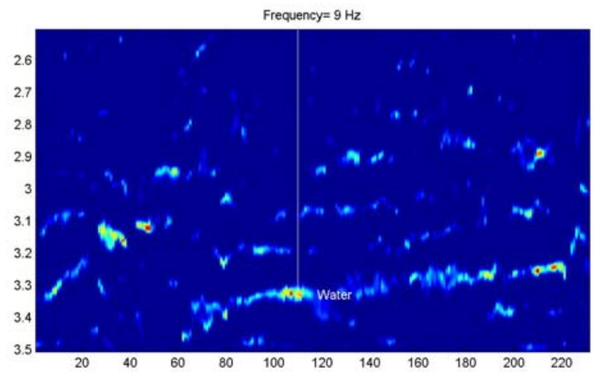
The second example we show here is an application of spiky decomposition to real seismic data, collected at an undisclosed location (below).

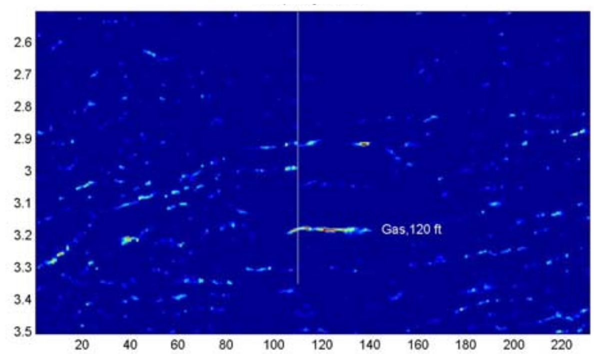




The figure above shows the sum of sparse spike decomposition results at all frequencies. Note the excellent horizontal continuity of the spiky coefficients. The decomposition reveals different details than can be seen in the original seismic section.

By displaying the panels with individual frequencies we reveal the anomalies that are attributable to different stratigraphic units within the section. The panel below shows the 24 Hz section, while the next panel below shows the 9 Hz section. The vertical white line shows the well that penetrated water at the level of low-frequency anomaly and the gas reservoir at the level of the 24 Hz anomaly.





Discussion

We would like to point out two particular aspects of the inverse decomposition. First, it is computationally expensive due to the iterative nature of conjugate gradient relaxation. Second, the exact utility of the method remains subjective. We merely state here that our decomposition methods produce pictures with different levels of detail, while acknowledging that much empirical research remains to be done, especially for 3-D cases.

References

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Acknowledgements

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